Documents

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Symmetries of differential equations in cosmology

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Abstract
The purpose of the current article is to present a brief albeit accurate presentation of the main tools used in the study of symmetries of Lagrange equations for holonomic systems and subsequently to show how these tools are applied in the major models of modern cosmology in order to derive exact solutions and deal with the problem of dark matter/energy. The key role in this approach are the first integrals of the field equations. We start with the Lie point symmetries and the first integrals defined by them, that is, the Hojman integrals. Subsequently, we discuss the Noether point symmetries and the well-known method for deriving the Noether integrals. By means of the Inverse Noether Theorem, we show that, to every Hojman quadratic first integral, it is possible to associate a Noether symmetry whose Noether integral is the original Hojman integral. It is emphasized that the point transformation generating this Noether symmetry need not coincide with the point transformation defining the Lie symmetry which produces the Hojman integral. We discuss the close connection between the Lie point and the Noether point symmetries with the collineations of the metric defined by the kinetic energy of the Lagrangian. In particular, the generators of Noether point symmetries are elements of the homothetic algebra of that metric. The key point in the current study of cosmological models is the introduction of the mini superspace, which is the space that is defined by the physical variables of the model, which is not the spacetime where the model evolves. The metric in the mini superspace is found from the kinematic part of the Lagrangian and we call it the kinetic metric. The rest part of the Lagrangian is the effective potential. We consider coordinate transformations of the original mini superspace metric in order to bring it to a form where we know its collineations, that is, the Killing vectors, the homothetic vector, etc. Then, we write the field equations of the cosmological model and we use the connection of these equations with the collineations of the mini superspace metric to compute the first integrals and subsequently to obtain analytic solutions for various allowable potentials and finally draw conclusions about the problem of dark energy. We consider the LCDM cosmological model, the scalar field cosmology, the Brans-Dicke cosmology, the f (R) gravity, the two scalar fields cosmology with interacting scalar fields and the Galilean cosmology. In each case, we present the relevant results in the form of tables for easy reference. Finally, we discuss briefly the higher order symmetries (the contact symmetries) and show how they are applied in the cases of scalar field cosmology and in the f (R) gravity. © 2018 by the author.

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